Lesson 8-1: Ratios and Proportions

Seventh grade math refresher

Most of this lesson is stuff you learned in 7^{th} grade math. Now I know that doesn't necessarily mean you remember it. But, it should come back pretty quickly.

Ratios

A <u>ratio</u> is a comparison between two values. Obviously to compare them they need to have the same unit of measurement. For instance you can't compare 2 miles to 3 quarts. But what about 83 inches to 7 feet? Duh you say…just convert the feet to inches. Yup! I'd agree…you can directly compare 83 inches to 84 inches (7 * 12).

There are a couple of common ways of representing a ratio. For a ratio of x to y we say:

$x : y$ or $\frac{x}{y}$

Ratio Example

A scale model of a car is 4 *in* long. The actual car is 15 *ft* long. What is the ratio of the length of the model car to the length of the real car?

This will be easiest to see in fraction form…don't forget to get common units!

$$\frac{4in}{15ft} = \frac{4in}{15ft \bullet 12in} = \frac{4in}{180in} = \frac{1}{45} \text{ or } 1:45$$

Proportions

A <u>proportion</u> is a statement that two ratios are equal. We express this as:

$\frac{a}{b} = \frac{c}{d}$ (and *sometimes* as $a : b = c : d$)

Properties of Proportions

There are four <u>properties of proportions</u> that are based on the properties of equality:

Property	**How to get it…**
1. $ad = bc$	$\frac{a}{b} = \frac{c}{d}$; $bd\left(\frac{a}{b}\right) = bd\left(\frac{c}{d}\right)$; $\cancel{b}d\left(\frac{a}{\cancel{b}}\right) = b\cancel{d}\left(\frac{c}{\cancel{d}}\right)$; $ad = bc$
2. $\frac{b}{a} = \frac{d}{c}$	$\frac{a}{b} = \frac{c}{d}$; $\frac{bd}{ac}\left(\frac{a}{b}\right) = \frac{bd}{ac}\left(\frac{c}{d}\right)$; $\frac{\cancel{b}d}{\cancel{a}c}\left(\frac{\cancel{a}}{\cancel{b}}\right) = \frac{b\cancel{d}}{a\cancel{c}}\left(\frac{\cancel{c}}{\cancel{d}}\right)$; $\frac{b}{a} = \frac{d}{c}$
3. $\frac{a}{c} = \frac{b}{d}$	$\frac{a}{b} = \frac{c}{d}$; $\frac{b}{c}\left(\frac{a}{b}\right) = \frac{b}{c}\left(\frac{c}{d}\right)$; $\frac{\cancel{b}}{c}\left(\frac{a}{\cancel{b}}\right) = \frac{b}{\cancel{c}}\left(\frac{\cancel{c}}{d}\right)$; $\frac{a}{c} = \frac{b}{d}$
4. $\frac{a+b}{b} = \frac{c+d}{d}$	$\frac{a}{b} = \frac{c}{d}$; $\frac{a}{b} + 1 = \frac{c}{d} + 1$; $\frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d}$; $\frac{a+b}{b} = \frac{c+d}{d}$

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Cross Product

Proportion property #1 is often referred as the "cross product:" $\frac{a}{b} \times \frac{c}{d}$; $ad = bc$

Proportion Examples

1. Complete the following statement: if $\frac{a}{4} = \frac{12}{b}$, then $\frac{b}{12} = \frac{?}{?}$.

   This is an application of proportion property #2…we've inverted the 2nd fraction.

   Thus the answer is the 1st fraction inverted: $\frac{4}{a}$

2. Solve the equation $\frac{2}{5} = \frac{n}{35}$

   First find the cross product, then solve for *n*:

   $\frac{2}{5} = \frac{n}{35}$; $2 \bullet 35 = 5 \bullet n$; $n = 14$

3. Solve the equation $\frac{x+1}{3} = \frac{x}{2}$

   Again, find the cross product, then solve for x:

   $\frac{x+1}{3} = \frac{x}{2}$; $2(x+1) = 3x$; $2x + 2 = 3x$; $x = 2$

Scale Drawings

The scale from a scale drawing tells you how the length of the drawing compares to the actual length. Usually the units in the drawing are different than the actual units. Makes sense doesn't it? If you are drawing a picture of a mountain, the drawing is going to be inches high…the mountain is likely thousands of feet high! For the mountain example a realistic scale may be 1 inch to 10,000 ft. Thus a drawing of Mt. Rainer would be about 15" tall in this scale.

Scale example

Two cities are 3½ *in* apart on a map with scale 1 *in* = 50 *mi*. Find the actual distance.

Every inch on the map represents 50 miles. So the 3.5 inches represents 3.5*50 or 175 *mi.*

Some 8th grade math – Quadratics

The standard form for a quadratic equation is $ax^2 + bx + c = 0, a \neq 0$. It is important to note that the equation is set equal to zero! When you can get a quadratic into this form it is very easy to solve if you use the quadratic formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where *a*, *b*, and *c* are the coefficients from the standard form.

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It is very important to be careful substituting the values into the equation. If you have negative values for any coefficient, be safe…put parenthesis around it to make sure you handle the negative signs right.

Solving a quadratic equation example

Solve for x: $-3x^2 - 5x + 5 = 4$

First note the quadratic is not equal to zero…we need to get it equal to zero first…

$$\begin{aligned} -3x^2 - 5x + 5 &= 4 \\ -4 &\quad -4 \\ -3x^2 - 5x + 1 &= 0 \end{aligned}$$

Now we identify the coefficients: a = -3, b = -5, and c = 1

Finally we plug these into the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-3)(1)}}{2(-3)} = \frac{5 \pm \sqrt{25+12}}{-6} = \frac{5 \pm \sqrt{37}}{-6}$$

$$\text{so } x = \frac{5+\sqrt{37}}{-6} \, (x \approx -1.85) \, and \;\; x = \frac{5-\sqrt{37}}{-6} \;\, (x \approx 0.18)$$

Homework Assignment

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